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MODEL OF FOURTH-ORDER CUMULANTS FOR DESCRIPTION OF TURBULENT TRANSPORT BY LARGE-SCALE VORTEX STRUCTURES

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A model for calculation of fourth-order cumulants is presented. Based on the Schwarz inequalities, a dependence between the coefficients of the model is established, which allows one to determine their numerical values. The use of an algebraic version of the model for parametrization of the process of turbulent diffusion in transport equations for third-order correlations does not require additional empirical information and offers a correct description of turbulent transport by large-scale vortex structures. Millionshchikov's hypothesis turns out to be insufficient for that.

Introduction. New directions of turbulent-flow investigations based on constructing closed equations of turbulent transport and their numerical implementation on computers have been intensely developed lately. The approach based on the use of a two-parametric model of turbulence gained wide application, as well as second-order closure models, which are effective from the computational viewpoint and yield results whose accuracy is sufficient for many practical applications. However, the use of these models for description of turbulent transport in stratified flows gives a qualitatively incorrect result in some cases (see, for example, [1]). The anisotropic character of the buoyancy effect on the structure of turbulence is manifested in the long-wave range of the spectrum of turbulent oscillations [2]. This spectral range corresponds to large-scale vortex structures (LVS) containing the main portion of turbulence energy. According to experimental and theoretical studies, the following LVS are formed in stratified flows: turbulent spots in the case of stable stratification and coherent structures in the case of unstable stratification, which are mainly responsible for turbulent transport. The effects of intermittency and asymmetry of vertical turbulent transport caused by the influence of LVS make the probability distributions of turbulent fluctuations significantly non-Gaussian. The turbulence structure in these flows is usually described by third-order closure models, where the triple correlations (asymmetry) are calculated from differential transport equations (see, for example, [3, 4]). A refinement of such models is proposed in the present paper.

Applicability of Millionshchikov's Quasinormality Hypothesis. Millionshchikov's quasinormality hypothesis for calculation of the fourth moments of statistical characteristics of turbulent flows (velocity fluctuations and mixed fourth-order covariations of velocity, temperature, and concentration fluctuations) is often used for closure in constructing semi-empirical models of turbulent transport of the second and third order. According to this hypothesis, fourth-order cumulants can be ignored in comparison with the corresponding correlation functions. As applied to the moments of hydrodynamic fields, this hypothesis means that we can use the equality that expresses the fourth moments through the second moments

$$C_{ijkl} = \langle u_i u_j u_k u_l \rangle - \langle u_i u_j \rangle \langle u_k u_l \rangle - \langle u_i u_j \rangle \langle u_k u_l \rangle - \langle u_i u_j \rangle \langle u_k u_l \rangle, \qquad (1)$$

where C_{ijkl} is the fourth-order cumulant of velocity fluctuations.

In some cases, the use of Eq. (1) leads to physically contradictory results [2] (for example, the appearance of negative portions of the spectrum of the turbulent kinetic energy [5, 6]). The latter circumstance

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is a consequence of the fact that, for given second and third moments, the probability distribution with the fourth cumulants equal to zero can be nonexistent; this circumstance is intimately connected with the fact that the Taylor series for the logarithm of the characteristic functional cannot be arbitrarily terminated.

In constructing semi-empirical models of turbulence, the use of Eq. (1) allows one to unite the terms that describe turbulent diffusion and generation of nonlinear interaction of the fluctuations in transport equations for triple correlations. The resultant first-order differential equations for the third moments do not take into account the necessary mechanism of attenuation of triple correlations. To take it into account, Craft et al. [4] and Deardorff [7] supplemented the equations by diffusion terms, and Andre et al. [3] used the clipping approximation of the triple correlations in accordance with the generalized Schwarz inequalities. In both cases, the procedure of taking into account the damping of the third moments seems physically incorrect. Hazen [6] showed that, for the description of the initial stage of turbulence generation in viscous incompressible fluid flows, the area of applicability of hypothesis (1) is limited by small amplitudes of fluctuations, such that the third moments are small, whereas the hypothesis of equality to zero of the fifth-order cumulants (under the condition of nonzero fourth-order cumulants) allows a significant expansion of the area of applicability of the model. A similar approach is used in the present paper. It allows one to obtain algebraic models for the fourth-order cumulants including a damping mechanism for triple correlations (upon substitution into differential transport equations for triple correlations). The models obtained do not require the use of additional empirical coefficients.

Model of Fourth-Order Cumulants of Velocity and Temperature. Theoretical and experimental studies indicate the formation of large-scale convective vortices (coherent structures) under conditions of unstable stratification in the planetary boundary layer (PBL) [1, 8]. These structures are mainly responsible for turbulent transport of momentum, heat, and substance. This transport has a countergradient character and cannot be physically correctly described by the existing second- and third-order closure models of turbulence. In particular, the models of triple correlations of the gradient type (taking into account the buoyancy effect among others [9, 10]) in the near-Earth layer yield a negative value of asymmetry of fluctuations of the vertical velocity component, which directly contradicts measurement data [11]; model [3], which contains differential transport equations for moments up to the third order inclusive, employs a physically incorrect procedure of clipping approximation.

Velocity fluctuations of large-scale convective vortices correspond to small values of the wave vector $\boldsymbol{k}$ in the spectrum of the turbulent kinetic energy (TKE). The use of Millionshchikov's hypothesis in the description of three-dimensional turbulence leads to the appearance of a negative region of the TKE spectrum at small $\boldsymbol{k}$ [2], which can be the reason for incorrectness of the models of turbulence in the description of turbulence transport by large-scale vortex structures.

To obtain a closed model of turbulent transport that does not imply equality to zero of the fourth-order cumulants, the closure procedure in the present paper is performed at the level of the fifth moments, i.e., it is assumed that the fifth-order cumulants are equal to zero. It should be noted that, according to Martsinkevich's theorem [12], the cumulant generation function cannot be a polynomial of power greater than two, i.e., either all the cumulants except for the first two are equal to zero (normal distribution), or there is an infinite number of cumulants other than zero. In the present paper, we assume that the equations for cumulants of order $n-2$, which are obtained by termination of the Taylor series of the generation function at the nth term, take into account the main physical mechanisms, and the error induced by them is insignificant. Thus, to obtain the distribution of the third-order cumulants (correlations), the latter are calculated from differential transport equations; the fourth-order cumulants are determined approximately (from algebraic expressions), and the fifth-order cumulants are assumed to be equal to zero, since their contribution is negligibly small. Results [13] of numerical simulation of vertical turbulent transport in a convective PBL verify the validity of this approach.

The equations for the second- and fourth-order moments of velocity in the Boussinesq approximation [1] have the form

$$\frac{\partial\langle u_iu_ju_ku_l\rangle}{\partial t}+U_m\frac{\partial\langle u_iu_ju_ku_l\rangle}{\partial x_m}=-\frac{\partial\langle u_iu_ju_ku_lu_m\rangle}{\partial x_m}-\sum_{ijkl}\Big[\langle u_iu_ju_ku_m\rangle\frac{\partial U_j}{\partial x_k}$$

$$+\langle u_iu_ju_k\rangle\frac{\partial\langle u_lu_m\rangle}{\partial x_m}+\beta g_m\langle u_iu_ju_k\theta\rangle\delta_{ml}+\frac{1}{\rho}\Big\langle u_iu_ju_k\frac{\partial p}{\partial x_l}\Big\rangle-\nu\Big\langle u_iu_ju_k\frac{\partial^2 u_l}{\partial x_m\,\partial x_m}\Big\rangle\Big], \qquad (2)$$

$$\frac{\partial\langle u_iu_j\rangle}{\partial t}+U_k\frac{\partial\langle u_iu_j\rangle}{\partial x_k}=-\frac{\partial\langle u_iu_ju_k\rangle}{\partial x_k}$$

$$-\sum_{ij}\Big[\langle u_iu_k\rangle\frac{\partial U_j}{\partial x_k}+\beta g_k\langle u_i\theta\rangle\delta_{jk}+\frac{1}{\rho}\Big\langle u_i\frac{\partial p}{\partial x_j}\Big\rangle-\nu\Big\langle u_i\frac{\partial^2 u_j}{\partial x_k\,\partial x_k}\Big\rangle\Big], \qquad (3)$$

where U_i and u_i are the mean and fluctuating components of the instantaneous velocity, p is the pressure fluctuations, $\beta = 1/\Theta$ is the volumetric expansion coefficient, Θ and θ are the mean and fluctuating potential temperature, g_m is the vector of acceleration of gravity, and ρ and ν are the density and viscosity of the fluid. In (2) and (3), we use a designation of the sum of the functions under the sign of summation $\sum_{ijkl}$, which differ in a cyclic permutation of the subscripts i, j, k, and l: $\sum_{ijkl} F(u_i,u_j,u_k,u_l,u_m) = F(u_i,u_j,u_k,u_l,u_m) + F(u_l,u_i,u_j,u_k,u_m) + F(u_k,u_l,u_i,u_j,u_m) + F(u_j,u_k,u_l,u_i,u_m)$.

To express the fifth-order moments in Eq. (2), we assume the fifth-order cumulants to be equal to zero:

$$C_{ijklm}=\langle u_iu_ju_ku_lu_m\rangle-\langle u_iu_j\rangle\langle u_ku_lu_m\rangle-\langle u_iu_k\rangle\langle u_ju_lu_m\rangle-\langle u_iu_l\rangle\langle u_ku_ju_m\rangle$$

$$-\langle u_iu_m\rangle\langle u_ku_lu_j\rangle-\langle u_ju_k\rangle\langle u_iu_lu_m\rangle-\langle u_ju_l\rangle\langle u_iu_ku_m\rangle-\langle u_ju_m\rangle\langle u_iu_ku_l\rangle$$

$$-\langle u_ku_l\rangle\langle u_iu_ju_m\rangle-\langle u_ku_m\rangle\langle u_iu_ju_l\rangle-\langle u_lu_m\rangle\langle u_iu_ju_k\rangle=0. \qquad (4)$$

Jovanovic and Durst [14] demonstrated the correctness of using Eq. (4) for the cumulants C_{33333} and C_{33331} in analyzing the statistical characteristics of a turbulent boundary layer measured on a flat plate. The equation for the fourth-order cumulants (simply cumulants in what follows) C_{ijkl} with account of (2)–(4) is

$$\frac{\partial C_{ijkl}}{\partial t}+U_m\frac{\partial C_{ijkl}}{\partial x_m}=\sum_{ijkl}\Big\{-C_{ijkm}\frac{\partial U_l}{\partial x_m}-C_{ijk\theta}\beta g_m\delta_{ml}-\frac{1}{\rho}\Big[\Big\langle u_iu_ju_k\frac{\partial p}{\partial x_l}\Big\rangle$$

$$-\langle u_iu_j\rangle\Big\langle u_k\frac{\partial p}{\partial x_l}\Big\rangle-\langle u_iu_k\rangle\Big\langle u_j\frac{\partial p}{\partial x_l}\Big\rangle-\langle u_iu_l\rangle\Big\langle u_j\frac{\partial p}{\partial x_k}\Big\rangle\Big]+\nu\Big[\Big\langle u_iu_ju_k\frac{\partial u_l}{\partial x_m\,\partial x_m}\Big\rangle$$

$$-\langle u_iu_j\rangle\Big\langle u_k\frac{\partial u_l}{\partial x_m\,\partial x_m}\Big\rangle-\langle u_iu_k\rangle\Big\langle u_j\frac{\partial u_l}{\partial x_m\,\partial x_m}\Big\rangle-\langle u_iu_l\rangle\Big\langle u_j\frac{\partial u_k}{\partial x_m\,\partial x_m}\Big\rangle\Big]$$

$$-\langle u_iu_ju_m\rangle\frac{\partial\langle u_ku_l\rangle}{\partial x_m}-\langle u_iu_m\rangle\frac{\partial\langle u_ju_ku_l\rangle}{\partial x_m}\Big\}-\langle u_iu_ku_m\rangle\frac{\partial\langle u_ju_l\rangle}{\partial x_m}-\langle u_ju_lu_m\rangle\frac{\partial\langle u_iu_k\rangle}{\partial x_m}, \qquad (5)$$

where $C_{ijk\theta} = \langle u_iu_ju_k\theta\rangle - \langle u_iu_j\rangle\langle u_k\theta\rangle - \langle u_iu_k\rangle\langle u_j\theta\rangle - \langle u_ju_k\rangle\langle u_i\theta\rangle$ is the mixed cumulant of velocity and potential temperature fluctuations. Equation (5) includes unknown cumulants (third and fourth terms on the right side of the equation). In the high-Reynolds-number approximation, the term with viscosity makes a negligibly small contribution and is assumed equal to zero in this paper. It follows from Eq. (5) that the mixed cumulant of velocity fluctuations and the derivative of pressure fluctuations tends to zero as the turbulence approaches the equilibrium state (Gaussian turbulence with zero cumulants of all orders higher than two). For parametrization of this cumulant, we use the assumption about the relaxation character of turbulence tendency to the equilibrium state. Then, the cumulant with pressure fluctuations can be represented as the relaxation term C_{ijkl}/τ_4 [$\tau_4 = \tau/C_4$, $\tau = E/\varepsilon$ is the characteristic time scale of turbulence, $E = 1/2\langle u_iu_i\rangle$ is the TKE density (per unit mass), ε is the spectral flux of the TKE (TKE dissipation rate), and C_4 is the factor of proportionality between the characteristic time scale of turbulence and the characteristic relaxation time of the cumulants]. In this case, Eq. (5) acquires the following form:

$$\frac{\partial C_{ijkl}}{\partial t} + U_m \frac{\partial C_{ijkl}}{\partial x_m} = \sum_{ijkl} \Big[- C_{ijkm} \frac{\partial U_l}{\partial x_m} - C_{ijk\theta}\beta g_m \delta_{ml} - \langle u_i u_j u_m \rangle \frac{\partial \langle u_k u_l \rangle}{\partial x_m}$$

$$- \langle u_i u_m \rangle \frac{\partial \langle u_j u_k u_l \rangle}{\partial x_m} \Big] - \langle u_i u_k u_m \rangle \frac{\partial \langle u_j u_l \rangle}{\partial x_m} - \langle u_j u_l u_m \rangle \frac{\partial \langle u_i u_k \rangle}{\partial x_m} - C_4 \frac{C_{ijkl}}{\tau}. \quad (6)$$

An algebraic expression for the cumulant C_{ijkl} is obtained from (6) in the steady case without account of the convective term:

$$C_{ijkl} = -\frac{\tau}{C_4} \Big\{ \sum_{ijkl} \Big[C_{ijkm} \frac{\partial U_l}{\partial x_m} + C_{ijk\theta}\beta g_m \delta_{ml} + \langle u_i u_j u_m \rangle \frac{\partial \langle u_k u_l \rangle}{\partial x_m}$$

$$+ \langle u_i u_m \rangle \frac{\partial \langle u_j u_k u_l \rangle}{\partial x_m} \Big] + \langle u_i u_k u_m \rangle \frac{\partial \langle u_j u_l \rangle}{\partial x_m} + \langle u_j u_l u_m \rangle \frac{\partial \langle u_i u_k \rangle}{\partial x_m} \Big\}. \quad (7)$$

In calculation of the evolution of the moments up to the third order inclusive, the use of parameterization (7) [the steady case of Eq. (6)] for the cumulant C_{ijkl} implies a faster relaxation of the latter as compared to the third moments. This condition imposes a constraint on the coefficient C_4: $C_4 > C_3$, where C_3 is the proportionality factor between the characteristic time scale of turbulence and the characteristic relaxation time of triple correlations (the coefficient of the correlation model with pressure fluctuations in equations for the third moments). To find C_4, we consider the Schwarz inequality for the triple correlation $\langle u_i^3 \rangle$

$$\langle u_i^2 \rangle \langle u_i^4 \rangle - \langle u_i^3 \rangle^2 \geqslant 0. \quad (8)$$

In [3], condition (8) for the correlation $\langle w^3 \rangle$ [using (1) for parametrization of the fourth-order moment $\langle w^4 \rangle$] was satisfied by clipping of the triple correlation $\langle w^3 \rangle$. In the present paper, we determine the upper boundary of the numerical value of the coefficient C_4 from the condition of satisfied inequality (8). We write a stricter inequality

$$C_{iiii} \geqslant \langle u_i^3 \rangle^2 / \langle u_i^2 \rangle, \quad (9)$$

and consider the process of relaxation attenuation of homogeneous turbulence in accordance with the laws

$$\langle w^2 \rangle = \langle w^2 \rangle_0 \exp(-t/\tau), \quad \langle w^3 \rangle = \langle w^3 \rangle_0 \exp(-C_3 t/\tau), \quad C_{iiii} = C_{iiii0} \exp(-C_4 t/\tau). \quad (10)$$

With account of (10), we obtain the following condition from (9): $C_4 \leqslant 2C_3 - 1$. To use correctly Eq. (7) for the cumulant C_{ijkl} [the steady case of Eq. (6)], we assume the numerical value of the coefficient C_4 to be equal to the upper boundary of the resultant condition: $C_4 = 2C_3 - 1$.

The algebraic model of the cumulant of velocity fluctuations (8) for a stratified flow includes mixed cumulants of velocity and potential temperature fluctuations. Using similar considerations, we can construct the following algebraic models for them:

$$C_{ijk\theta} = -\frac{\tau}{C_{4\theta}} \Big\{ \sum_{ijk} \Big[C_{ijm\theta} \frac{\partial U_k}{\partial x_m} + C_{ijkm} \frac{\partial \Theta}{\partial x_m} + C_{ij\theta\theta}\beta g_m \delta_{km} + \langle u_i u_j u_m \rangle \frac{\partial \langle u_k \theta \rangle}{\partial x_m}$$

$$+ \langle u_i \theta u_m \rangle \frac{\partial \langle u_j u_k \rangle}{\partial x_m} + \langle u_i u_m \rangle \frac{\partial \langle u_j u_k \theta \rangle}{\partial x_m} \Big] + \langle \theta u_m \rangle \frac{\partial \langle u_i u_j u_k \rangle}{\partial x_m} \Big\}; \quad (11)$$

$$C_{ij\theta\theta} = -\frac{\tau}{C_{4\theta}} \Big\{ \sum_{ij} \Big[C_{im\theta\theta} \frac{\partial U_j}{\partial x_m} + 2C_{ijm\theta} \frac{\partial \Theta}{\partial x_m} + C_{i\theta\theta\theta}\beta g_m \delta_{jm} + \langle \theta u_m \rangle \frac{\partial \langle u_i u_j \theta \rangle}{\partial x_m}$$

$$+ 2\langle u_i \theta u_m \rangle \frac{\partial \langle u_j \theta \rangle}{\partial x_m} + \langle u_i u_m \rangle \frac{\partial \langle u_j \theta^2 \rangle}{\partial x_m} \Big] + \langle \theta^2 u_m \rangle \frac{\partial \langle u_i u_j \rangle}{\partial x_m} + \langle u_i u_j u_m \rangle \frac{\partial \langle \theta^2 \rangle}{\partial x_m} \Big\}; \quad (12)$$

$$C_{i\theta\theta\theta} = -\frac{\tau}{C_{4\theta}} \Big[C_{m\theta\theta\theta} \frac{\partial U_i}{\partial x_m} + 3C_{im\theta\theta} \frac{\partial \Theta}{\partial x_m} + C_{\theta\theta\theta\theta}\beta g_m \delta_{im} + 3\langle \theta^2 u_m \rangle \frac{\partial \langle u_i \theta \rangle}{\partial x_m}$$

$$+ 3\langle u_i u_m \theta \rangle \frac{\partial \langle \theta^2 \rangle}{\partial x_m} + 3\langle \theta u_m \rangle \frac{\partial \langle u_i \theta^2 \rangle}{\partial x_m} + \langle u_i u_m \rangle \frac{\partial \langle \theta^3 \rangle}{\partial x_m} \Big]; \quad (13)$$

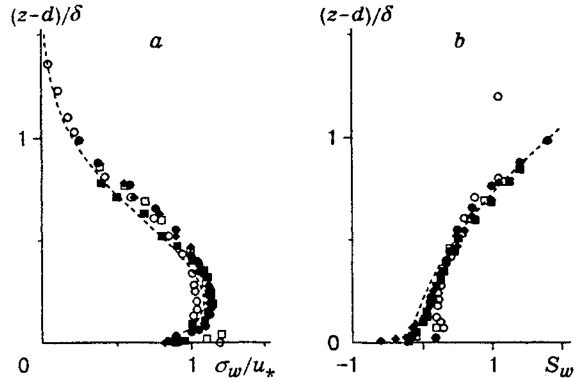

Fig. 1

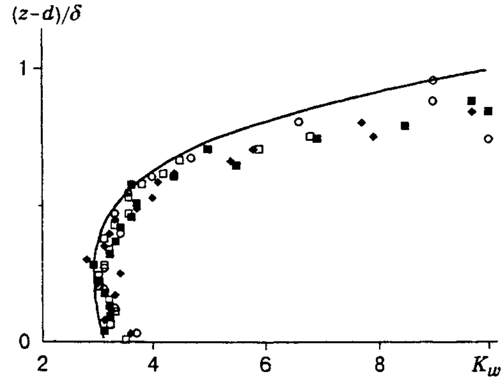

Fig. 2

$$C_{\theta\theta\theta\theta} = -\frac{\tau}{C_{4\theta\theta}}\left[4C_{m\theta\theta\theta}\frac{\partial\Theta}{\partial x_m} + 6\langle u_m\theta^2\rangle\frac{\partial\langle\theta^2\rangle}{\partial x_m} + 4\langle u_m\theta\rangle\frac{\partial\langle\theta^3\rangle}{\partial x_m}\right]. \quad (14)$$

Since the mechanism of relaxation attenuation in (11)–(13) is caused by correlations with pressure fluctuations, and in Eq. (14) (as in equations for the correlations $\langle\theta^2\rangle$ and $\langle\theta^3\rangle$) it is caused by molecular heat transfer, the relaxation coefficients $C_{4\theta}$ and C_θ for Eqs. (11)–(13) and (14) are not assumed to be equal.

Stricter Schwarz inequalities for the cumulants $C_{ii\theta\theta}$ and $C_{\theta\theta\theta\theta}$ have the form

$$C_{ii\theta\theta} \geqslant \frac{\langle u_i^2\theta\rangle^2}{\langle u_i^2\rangle}, \qquad C_{\theta\theta\theta\theta} \geqslant \langle\theta^3\rangle^2/\langle\theta^2\rangle. \quad (15)$$

From inequalities (15), in considering the process of heat propagation in the field of decaying turbulence, we find the equations for the relaxation coefficients $C_{4\theta} = 2C_{3\theta\theta} - 1$ and $C_\theta = 2C_{3\theta} - r$, where $C_{3\theta\theta}$ and $C_{3\theta}$ are the coefficients of the characteristic time scales of relaxation of the triple correlations $\langle u_i^2\theta\rangle$ and $\langle\theta^3\rangle$, respectively, and $r = \tau/\tau_\theta$ is the ratio of the time scales of velocity and temperature fluctuations ($\tau_\theta = \langle\theta^2\rangle/\varepsilon_\theta$, where ε_θ is the destruction of turbulent fluctuations of the temperature).

In modeling the evolution of a convective PBL, Ilyushin and Kurbatskii [13] used model (7), (11) with ignored buoyancy effects for parametrization of the processes of turbulent diffusion in transport equations for the triple correlations $\langle w^3\rangle$ and $\langle w^2\theta\rangle$. The calculated profiles of the vertical TKE flux $\langle wE'\rangle$ and the correlation $\langle w^3\rangle$, which are positive over the entire PBL height, are in good agreement with the measurement data. Nevertheless, the lack of distributions of the fourth-order moments measured in the PBL does not allow us to evaluate the adequacy of the proposed model for (7) and (11)–(14) cumulants.

Raupach [15] measured the distributions of dispersion σ_w ($\sigma_w^2 = \langle w^2\rangle$), asymmetry $S_w = \langle w^3\rangle/\langle w^2\rangle^{3/2}$, and kurtosis $K_w = \langle w^4\rangle/\langle w^2\rangle^2$ of the vertical velocity in the boundary layer on a rough flat plate. However, since there are no data on the vertical distribution of the characteristic time scale of turbulence τ (or spectral TKE flux) in [15], it is not possible to verify the adequacy of the proposed model for cumulants by immediate substitution of the measured quantities into Eq. (8). Nevertheless, taking into account that the behavior of the third-order moment $\langle w^3\rangle$ for the flow mentioned is described by an algebraic model of the gradient type obtained from an appropriate differential transport equation in the local balance approximation [16]

$$\langle w^3\rangle = (-3\tau/C_3)\langle w^2\rangle\partial\langle w^2\rangle/\partial z,$$

the value of the cumulant can be found from the following algebraic expression obtained from (7):

$$C_{3333} = \frac{C_3/3}{2C_3 - 1}\left[\frac{\langle w^3\rangle^2}{\langle w^2\rangle} + 4\langle w^3\rangle\frac{\partial\langle w^3\rangle/\partial z}{\partial\langle w^2\rangle/\partial z}\right]. \quad (16)$$

Here $C_3 = 4$ [16].

Figure 2 shows the kurtosis profile K_w calculated by substituting into (16) analytical functions, which describe the distributions of dispersion σ_w and asymmetry S_w of vertical fluctuations of the velocity w (curves in Fig. 1) measured in [15] (d is the height of displacement of the fluid by roughness elements and δ is the boundary-layer thickness). The points in Figs. 1 and 2 indicate measurement data [15]. It is seen from Fig. 2 that algebraic model (7) for the cumulant C_{3333} correctly describes its behavior in a flat-plate boundary layer.

The results presented and the results [13] allow us to conclude that the algebraic model for cumulants can be used in boundary layers both to parametrize the processes of turbulent diffusion in transport equations for the third-order moments and to calculate the cumulants themselves.

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